1. **Number Sense and Operation** – A student, applying reasoning and problem solving, will use number sense and operations to represent numbers in multiple ways, understand relationships among numbers and number systems, make reasonable estimates, and compute fluently within a variety of relevant cultural contexts, including those of Montana American Indians.

2. **Data Analysis Mathematics** – A student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability, statistics, and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts, including those of Montana American Indians.

3. **Geometric Reasoning** – A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant and cultural contexts, including those of Montana American Indians.

4. **Algebraic and Functional Reasoning** – A student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables and will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts, including those of Montana American Indians.

**Mathematical rigor** is an elusive term with multiple meanings. To a pure mathematician, rigor is a mark of excellence. To a K-12 educator, “rigorous” often means “difficult,” as in “AP calculus is rigorous.” In the Montana Mathematics Content Standards, **rigor** is a process where students:

* approach mathematics with a disposition to accept challenge and apply effort;
* engage in mathematical work that promotes deep knowledge of content, analytical reasoning, and use of appropriate tools; and
* emerge fluent in the language of mathematics, proficient with the tools of mathematics, and empowered as mathematical thinkers.
Mathematical Processes, Proficiency, and Principles

Mathematical Processes

The National Council of Teachers of Mathematics Principles and Standards for School Mathematics recognizes five processes that complement and enhance the learning of mathematical content: connections, communication, representation, problem solving, and reasoning. The Office of Public Instruction (OPI) advocates the importance of viewing mathematics through these five lenses because:

- **Mathematics does not exist in isolation.** Learning takes place when students see connections within mathematics and apply their mathematical knowledge to other disciplines and authentic contexts;
- **Mathematics does not follow a single fixed path.** Learning takes place through multiple routes as students visualize, represent, interpret, and construct mathematical ideas in a variety of ways;
- **Mathematics is not a private enterprise.** Learning takes place when students express their mathematical ideas both verbally and in writing, engage in discourse and work together to build concepts;
- **Mathematics is not free of context.** Learning takes place when students use mathematics to explore ideas, model situations, solve problems, and question and comprehend the world around them; and
- **Mathematics is about doing, not simply knowing.** Learning takes place when students reason, conjecture, reflect, predict, and justify their thinking to themselves and others.

Mathematical Proficiency

The National Research Council has identified five research-based building blocks for mathematical proficiency. These are:

- **Conceptual understanding** - comprehension of mathematical concepts, operations, and relations;
- **Procedural fluency** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- **Strategic competence** - ability to formulate, represent, and solve mathematical problems;
- **Adaptive reasoning** - capacity for logical thought, reflection, explanation, and justification; and
- **Productive disposition** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (National Research Council 116)
## Mathematical Processes, Proficiency, and Principles

### Principles of Montana Mathematics

The Montana Mathematics Content Standards were conceived and developed under a set of guiding principles agreed upon by all stakeholders in the process. Through high-quality professional development, teachers must embrace these principles and embed them into curriculum planning, instruction, and assessment of mathematics.

- **All students can successfully learn mathematics.** Adopting this view requires teachers to hold high expectations for all their students and to create mathematical experiences that enable success for all.

- **Mathematical processes are fundamental companions to content.** The five processes described earlier are essential to creating an environment where students can acquire, apply, and make meaning of mathematics.

- **Mathematics is a human endeavor with scientific, social, and cultural relevance.** Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians.

- **Technology is integral to learning mathematics.** Today's students are fluent in the language of digital media and technology. Montana educators must maximize technology's potential for enhancing mathematics learning.

- **Mathematics education is for the future, not for today.** To paraphrase a now-famous quote from Karl Fisch (qtd. in *Shift Happens*) today's students are preparing for jobs that do not yet exist, using technologies that are yet to be invented, to solve problems yet to be identified. Mathematics must be viewed not only through the lens of past experience, but also through a lens that will steer our students through the 21st century.
# Billings Public Schools-Geometry Standards and Learning Objectives

**Number Sense and Operation**

A student, applying reasoning and problem solving, will use number sense and operations to represent numbers in multiple ways, understand relationships among numbers and number systems, make reasonable estimates, and compute fluently within a variety of relevant cultural contexts, including those of Montana American Indians.

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<thead>
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| **A. Quantification**: Use multiple notations to perform and interpret the effects of operations on very large and very small numbers with and without technology. | 1. Identify and explain situations where an estimate is adequate and appropriate. (E, A)  
2. Use estimated values of irrational numbers to check for reasonableness of solution to problems. (E, A)  
3. Round solutions and choose the appropriate precision to report the answer based on the context of the problem. (E, A) | precision, error |
| **B. Estimation and Accuracy**: Identify situations where estimation is appropriate and determine the degree of accuracy needed for a given problem situation (and the appropriate precision in which to report answers). | 4. Express a real number in a variety of equivalent forms using exponents, radicals, or irrational numbers. (A)  
5. Simplify real number expressions containing rational exponents, radicals or irrational numbers. (A)  
6. Add, subtract, multiply and factor polynomials (A) | exponent, radical, irrational number, real number, rational number |
| **C. Equivalence with Multiple Notation**: Given a representation of a number or expression, find equivalent representations using multiple notations (e.g., $x^{1/2}$ vs. $\sqrt{x}$ and visual representation of multiplying binomials) | 7. Apply real-number properties to solve problems. (A) | |
| **D. Properties of Numbers and Number Systems**: Analyze and apply the properties of numbers and number systems. | | |
| **E. Modeling Relationships and Change**: Identify givens and unknowns in familiar and unfamiliar situations (e.g., finance, culture, including Montana American Indians, and nature) and describe relationships between variables | | |
Billings Public Schools-Geometry Standards and Learning Objectives

**Data Analysis Mathematics**  A student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability, statistics, and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts, including those of Montana American Indians.

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<td><strong>A. Representing and Analyzing Data:</strong></td>
<td>Select, create, and compare graphical or numerical representations of data sets using technology when appropriate. Reason about distributions using measures of central tendency and spread (e.g., percentiles, quartiles, inter-quartile range, and standard deviation).</td>
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<td><strong>B. Evaluating Validity:</strong></td>
<td>Evaluate the validity of reports based on collected and/or published data by considering the source of the data, the design of the study, and the way data are displayed, analyzed, and interpreted.</td>
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<td><strong>C. Rules of Probability and Expected Value:</strong></td>
<td>Make, evaluate, and justify decisions based on probabilities in multicultural situations, including those of Montana American Indians (e.g., finding expected value and using rules of probability).</td>
<td>8. Make, evaluate, and justify decisions based on probabilities. (A)</td>
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<tr>
<td><strong>D. Counting Methods:</strong></td>
<td>Use technology as needed to determine the possible number of outcomes for an event or compound event using the fundamental counting principle, permutations, combinations, and other systematic counting methods.</td>
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<tr>
<td><strong>E. Curve Fitting:</strong></td>
<td>Model two-variable data using curve fitting with and without technology. Write an equation for a given model and decide when or if predictions based on this equation are valid.</td>
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Billings Public Schools-Geometry Standards and Learning Objectives

**Geometric Reasoning** A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant and cultural contexts, including those of Montana American Indians.

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| **A. Conjectures and Inductive Reasoning:** Formulate and evaluate conjectures about geometric objects and their properties, with and without technology, applying inductive reasoning when appropriate. | 9. Formulate and evaluate conjectures based on geometric properties including inscribed angles, radii, and chords. (I, D, E, A)  
10. Apply deductive and inductive reasoning to solve geometric problems. (I, D, E, A) | chord, conjecture, inductive reasoning, deductive reasoning |
| **B. Applications of Geometric Models:** Use spatial reasoning and geometric models to solve problems with and without technology in the contexts of art, science, and culture, including Montana American Indians. | 11. Build and create isometric drawings, foundational drawings, and orthographic drawings of geometric solids and their views. (I, D, E, A)  
12. Visualize and draw three-dimensional objects from different perspectives and identify cross sections. (I, D, E, A)  
13. Construct geometric representations with and without technology. (I, D, E, A)  
14. Use geometric models and spatial reasoning to solve problems in the context of art, science, and culture, including those of Montana American Indians. (I, D, E, A) | cross sections, compass, straight edge, tangent, inscribe, circumscribe |
| **C. Multiple Geometric Approaches:** Identify, analyze, and use transformational, coordinate, and synthetic geometric approaches to solve problems. | 15. Perform transformations of shapes in a plane with and without the coordinate system. (I, D, E, A)  
16. Model and describe simple transformations and their compositions (e.g., composition of two reflections over parallel lines produces a translation). (I, D, E, A)  
17. Use specific geometric approaches to solve problems (e.g., computer applications, physical models). (I, D, E, A)  
18. Verify properties of dilations given by a center and a scale factor. (I, D, E, A)  
19. Determine lengths of segments using the distance formula, find midpoints, and determine whether lines are parallel or perpendicular using slopes. (E, A) | composition of transformations, distance around a circular arc |

I-Introduce, D-Develop, *E-Essential*, A-Apply
**Billings Public Schools-Geometry Standards and Learning Objectives**

**Geometric Reasoning**  
A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant and cultural contexts, including those of Montana American Indians.

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| **D. Indirect Measurement:** Determine measures of two- and three-dimensional objects and their elements using trigonometric ratios, proportionality, the Pythagorean Theorem, and angle relationships. | 20. Find sides and angles of right triangles using the Pythagorean Theorem, proportionality and trigonometric ratios. (**E**, **A**)  
21. Use appropriate strategies to find the measures of length, perimeter, area, surface area and volume of composite figures and solids (e.g., pyramids, cones). (**I**, **D**, **E**, **A**)  
22. Use the law of sines and cosines to find indirect measurements in on-right triangles. (**I**)  
23. Derive the equation of a circle of given center and radius using the Pythagorean Theorem to find the center and radius of a circle given by an equation. (**I**, **D**, **E**, **A**) | sine, cosine, tangent, composite figure |
| **E. Methods of Proof:** Establish the validity of geometric conjectures using deductive reasoning, indirect proof, and counterexamples, and critique arguments made by others. | 24. Use deductive reasoning, indirect proof and counterexamples to justify geometric conjectures and properties. (**I**, **D**)  
25. Make and evaluate logical arguments justifying properties of figures on the coordinate plane (e.g., triangles and quadrilaterals). (**I**, **D**, **E**, **A**)  
26. Make and evaluate logical arguments justifying congruency and similarity of figures (e.g., triangles and quadrilaterals). (**I**, **D**, **E**, **A**)  
27. Prove theorems about properties of parallel lines cut by a transversal and perpendicular bisectors. (**I**, **D**, **E**, **A**) | deductive reasoning, proof format ("T"/two column proof, flow, paragraph, algebraic), indirect proof, counterexample, equidistant |
A student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables and will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts, including those of Montana American Indians.

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<td>A. Representing Functions: Represent functions in a variety of ways including tables, graphs or diagrams, verbal descriptions, and symbolic expressions in recursive and explicit form. Justify the choice of an appropriate form for solving a given problem.</td>
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<td>B. Variables and Parameters: Determine the appropriate symbolic representation of a given contextual situation (e.g., variables and parameters in equations, inequalities, functions, and matrices).</td>
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<tr>
<td>C. Solving Systems of Equations and Inequalities: Solve a variety of equations, inequalities and systems of equations and inequalities, justify the solution process, and interpret the solution in context.</td>
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<td>D. Families of Functions and Transformations: Analyze the effects of transformations on families of functions and recognize their characteristics. Represent and use functions in equivalent forms to identify and perform transformations.</td>
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<tr>
<td>E. Analyzing and Conjecturing with Models: Given data or a problem situation, select and use an appropriate function model to analyze results or make a prediction with and without technology using cultural contexts, including those of Montana American Indians.</td>
<td>28. Decide if a solution is reasonable in the context of the original situation. (A)</td>
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## Billings Public Schools-Geometry Standards and Learning Objectives

### Works Cited